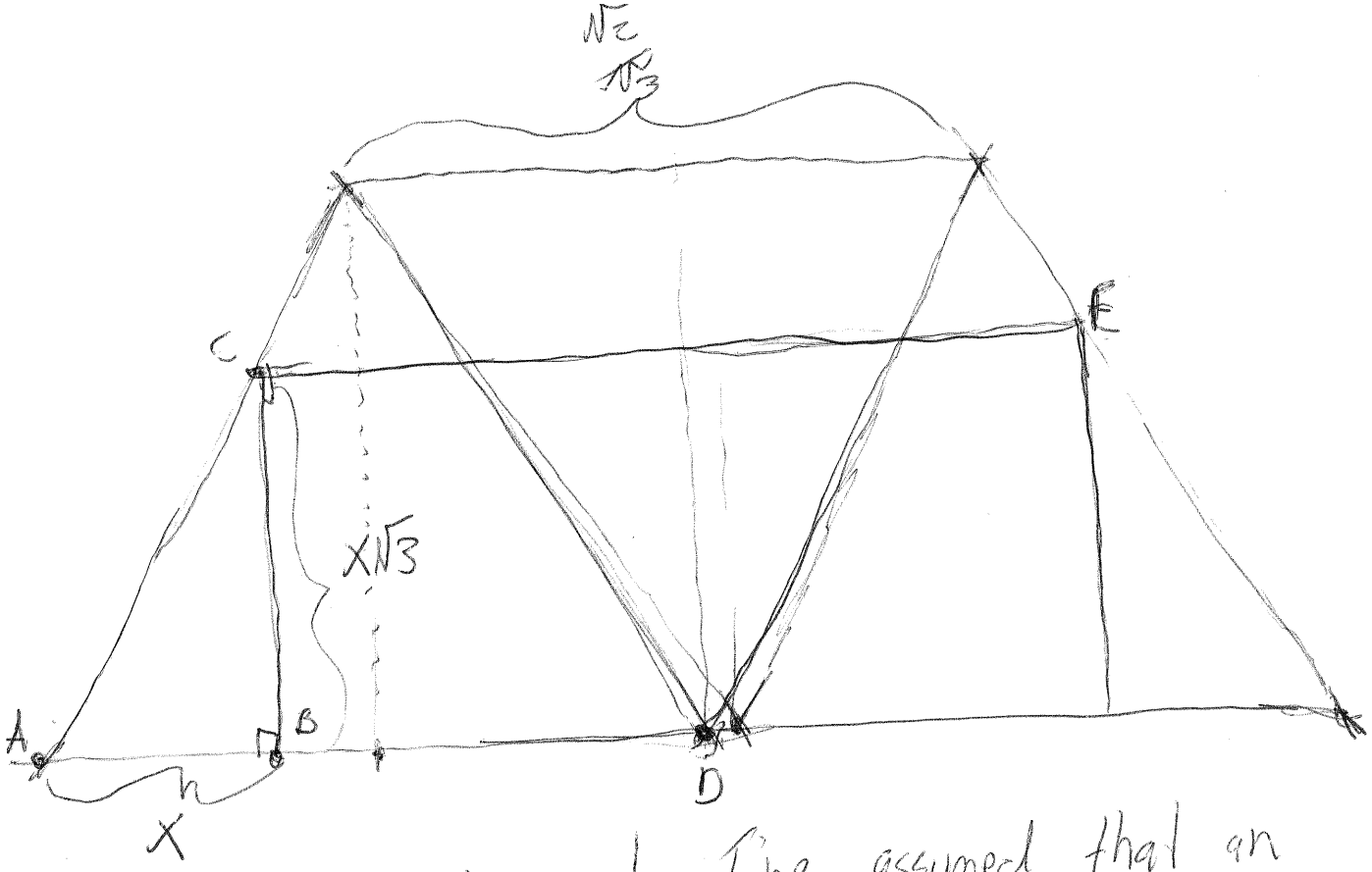

$\overline{AB} = x$, $\overline{BC} = x\sqrt{3}$, $2(\overline{CB}) = \overline{CE}$

$\overline{BD} = \frac{\sqrt{2}}{\sqrt{3}} - x$

we want $\overline{BC} = \overline{BD}$

$\frac{\sqrt{2}}{\sqrt{3}} - x = x\sqrt{3}$

$x(1+\sqrt{3}) = \frac{\sqrt{2}}{\sqrt{3}}$

$x = \frac{\sqrt{2}}{3+\sqrt{3}}$

$\overline{CE} = 2(x\sqrt{3}) = 2\left(\frac{\sqrt{6}}{3+\sqrt{3}}\right)$

$(\overline{CE})^2 = \frac{24}{9+6\sqrt{3}+3} = \frac{4}{2+\sqrt{3}} \approx 1.0718$

I've assumed that an edge of the square is parallel to the edge of the Hexagon. I let x determine the shape of a rectangle parallel to the Hexagon and solved for x when that rectangle was a square.

I get a different answer than the "Cheat" page.